PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 08

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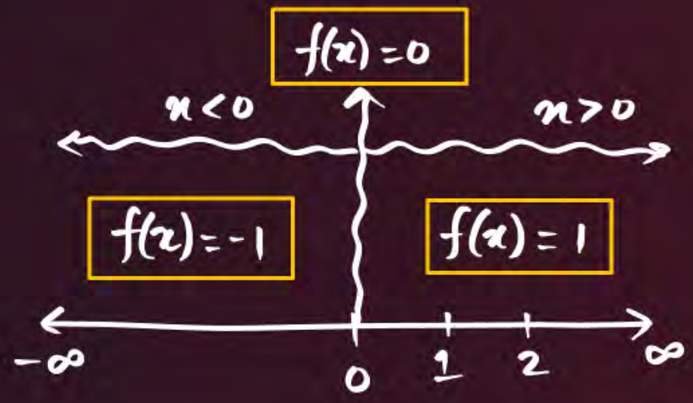
Today's Targets

1. PYQs ✓
2. Doubts ✓
3. _____
4. _____

Question

Show that the Signum function $f: R \to R$, (R: Codomain) given by

$$f(x) = \begin{cases} 1, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$$ is neither one-one nor onto.



Sol:

One-Oneness

$f(1) = 1$, $f(2) = 1$

$\Rightarrow f(1) = f(2)$

$\Rightarrow$ f is not one-one

Ontoness

Let $y = 2 \in R$

There is no such value of $x \in R$

Such $y = f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

$\Rightarrow$ f is not onto.

$f(3)$ → $2(3)+1 = 7$ ✗

→ $(3)+2 = 5$ ✓

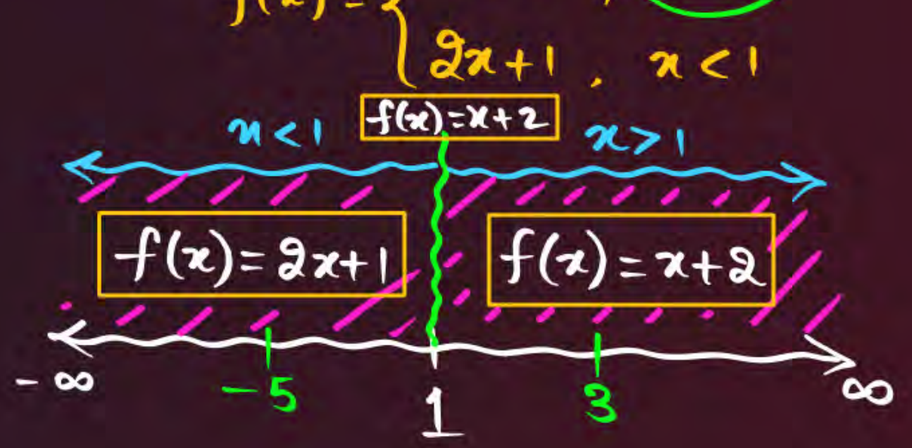
$f(-5)$ → $2(-5)+1 = -9$ ✓

→ $(-5)+2 = -3$ ✗

$f(1) = (1)+2 = 3$

$$f(x) = \begin{cases} x+2, & x \geq 1 \\ 2x+1, & x < 1 \end{cases}$$

$x=1$ or $x>1$



Question (Imp)

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ (B → codomain.) defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Sol: **One-One ness**

$f(a) = f(b)$, where $a, b \neq 3$

$$\frac{a-2}{a-3} = \frac{b-2}{b-3}$$

$$(a-2)(b-3) = (b-2)(a-3)$$

$$ab - 3a - 2b + 6 = ab - 3b - 2a + 6$$

$$-3a + 2a = -3b + 2b$$

$$-a = -b$$

$$a = b$$

$\Rightarrow$ f is one-one.

Ontoness

Let $y \in B = R - \{1\}$

Such that $y = \frac{x-2}{x-3}$, $x \neq 3$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow xy - x = -2 + 3y$$

$$\Rightarrow x(y-1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

Since, $y \in R - \{1\}$

$y - 1 \in R - \{0\}$ and $3y - 2 \in R$

$$\Rightarrow \frac{3y-2}{y-1} \in R$$

$$\Rightarrow x \in R - \{3\}$$

$$\Rightarrow x \in A$$

$\Rightarrow$ f is onto

Question

What is the range of the function $f(x) = \frac{|x-1|}{x-1}, x \neq 1$?

Ans: Range = $\{1, -1\}$

$$\frac{|x-1|}{x-1} = \begin{cases} +\frac{(x-1)}{x-1}, & x > 1 \\ -\frac{(x-1)}{x-1}, & x < 1 \end{cases}$$

$$f(x) = \frac{|x-1|}{x-1} = \begin{cases} +1, & x > 1 \\ -1, & x < 1 \end{cases}$$

Range = $\{-1, 1\}$

$$|x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x-1| = \begin{cases} +(x-1), & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$$

$$|x+2| = \begin{cases} +(x+2), & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$$

Question

Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N: y = 4x + 3, x \in N\}$. Show that f is bijective.

$Y = \{7, 11, 15, \ldots\}$
↳ Range

Sol:

One-one ness

$f(a) = f(b)$

$4a + 3 = 4b + 3$

$4a = 4b$

$a = b$

$\Rightarrow f$ is one-one

Ontoness

Since, Y is the codomain as well as Range of f

$\Rightarrow$ Codomain = Range

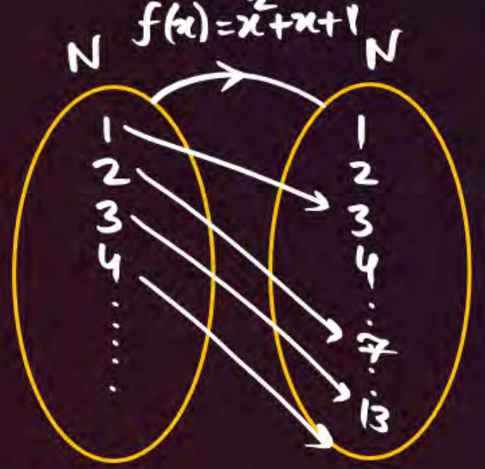
$\Rightarrow f$ is onto

Question

$N = \{1, 2, 3, 4 \ldots\}$

Prove that the function $f: N \to N$ (A = N, B = N) is defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Sol: One-One ness



$f(1) = 1+1+1 = 3$

$f(2) = 4+2+1 = 7$

$f(3) = 9+3+1 = 13$

$\Rightarrow$ f is one-one

Ontoness

Let $y = 1 \in N$

such that $y = x^2 + x + 1$

$\Rightarrow 1 = x^2 + x + 1$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x(x+1) = 0$

$\Rightarrow x = 0 \notin N, \; x = -1 \notin N$

$\Rightarrow$ f is not onto.

Ontoness.

for every $y \in B$ there exists at least one $x \in A$ such that $y = f(x)$

for $y \in B \Rightarrow x \in A$

Question

W = whole No. = {0, 1, 2, 3, 4, ... }

* Imp.

Prove that the function $f: W \to W$ is defined by $f(x) = \begin{cases} x-1, & x \text{ is odd} \\ x+1, & x \text{ is even} \end{cases}$ is bijective.

Sol: One-One ness

Case 1. a is odd and b is odd

$f(a) = a-1$, $f(b) = b-1$

let $f(a) = f(b)$

$a-1 = b-1$

$a = b$ $\Rightarrow$ f is one-one

Case 2: a is even, and b is even

$f(a) = a+1$, $f(b) = b+1$

let $f(a) = f(b)$

$a+1 = b+1$

$a = b$ $\Rightarrow$ f is one-one

Case 3. a is odd and b is even

$f(a) = \underbrace{a-1}_{\text{even}}$, $f(b) = \underbrace{b+1}_{\text{odd}}$

let $f(a) = f(b)$

↳ condition not applicable

$\Rightarrow$ f is one-one

Ontoness

Case 1. let y is even $\Rightarrow y = 2k \in W$

$2k = x-1 \Rightarrow x = 2k+1 \in W$

$\Rightarrow$ f is onto

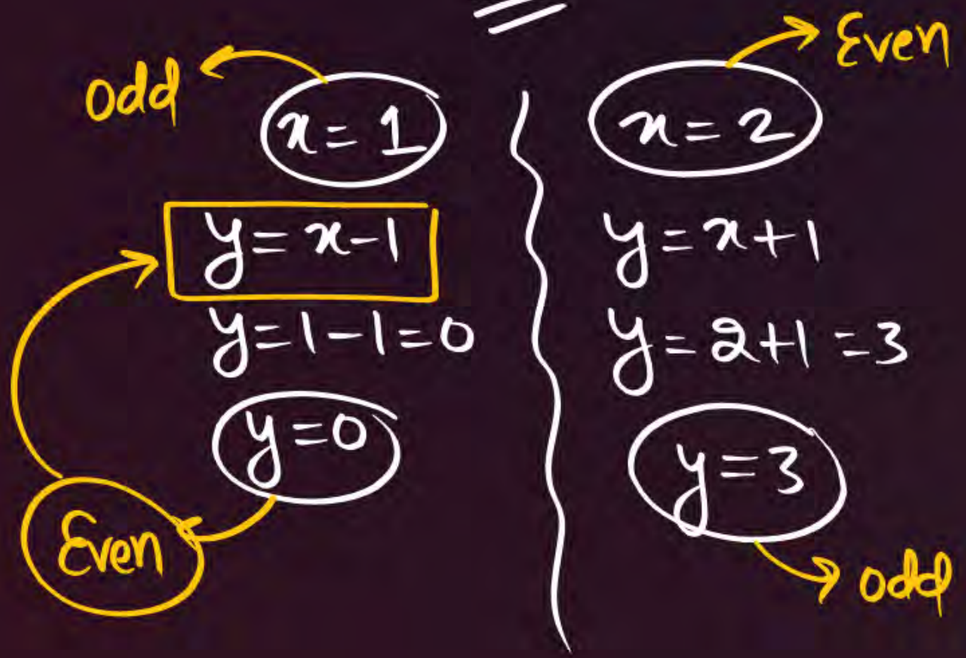
Case 2. let y is odd $\Rightarrow y = 2k+1 \in W$

$2k+1 = x+1 \Rightarrow x = 2k \in W$

$\Rightarrow$ f is onto.

$$f(x)=\begin{cases} x-1, & x \text{ is odd} \\ x+1, & x \text{ is even} \end{cases}$$

Ex.



let $y = 2k \in w$

$x - 1 = 2k$

$x = 2k + 1 \in w$

$\Rightarrow$ f is onto

let $y = 2k + 1 \in w$

$x + 1 = 2k + 1$

$x = 2k \in w$

$\Rightarrow$ f is onto

Question (Homework)

Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall\, x \in R$ is neither one-one nor onto.

Hints.

↳ one-one Mathematically prove karna

↳ onto Mathematically prove karna

THANK
YOU